TCRA : A Time-based Channel Reservation Scheme for Handover Requests in LEO Satellite Systems

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Abstract—**In this paper, we propose a Time-based Channel Reservation Algorithm (TCRA) suitable for handover and call admission control procedures in future mobile satellite systems. These systems are characterized by a high rate of handover attempts which can degrade significantly their performance. Therefore, we propose TCRA, a scheme which guarantees a null handover failure probability by using a channel reservation strategy in the cells to be crossed by the user. The performance of TCRA has been compared to the Guaranteed Handover (GH) scheme. The TCRA reservation method has the advantage of a better channel utilization by locking the resources only for their expected time of use. A mathematical model has been developed for both schemes, and its results have been validated through simulations.**

I. Introduction

The mobile telecommunication market knows a never-predicted growth. Some mobile service operators are expanding their networks and others are studying new solutions based on satellite links supporting either narrow or large band services. Some of the proposed solutions are based on geostationary (GEO) satellites equipped with simple on-board processing and switching facilities; other ones propose the use of Low and Medium Earth Orbit satellites (LEO, MEO). The Non-GEO satellite systems have the ability to provide large coverage areas and constitute an ideal solution for the support of multicast applications [6], [8], [1]. However, these systems are characterized by a dynamic network topology which leads, at a user level, to a high number of handover attempts. This problem should be alleviated by implementing new call admission and handover control techniques for a better QoS performance.

Several approaches for handover prioritization proposed in terrestrial cellular systems have been studied for mobile satellite networks. These approaches include the guard channel scheme [13], handover queueing [4], [5], and connection admission control algorithms [10], [14], [7].

Our proposed TCRA (Time-based Channel Reservation Algorithm) scheme exploits the fact that the relative motion of the users (either fixed or mobile) is predictable. Therefore, TCRA anticipates the users motion and reserves resources accordingly. It estimates the residence time of the user in each cell to be crossed and reserve a resource during the corresponding residence time interval.

The remaining of the paper is organized as follows. In section II, we introduce the handover issue in the special context of satellite constellations. We also present a brief description of the GH (Guaranteed Handover) scheme [12], [9] and highlight its main drawbacks. Section III describes the TCRA scheme in details and show how TCRA improves the GH performance. An analytical approach is developed in section IV to derive the performance of TCRA and GH algorithms. Simulation experiments, contained in section V, are derived to validate the results obtained analytically. Finally, we report the conclusions in section VI.

II. Handover problem in a LEO satellite context

In future LEO satellite systems, the footprint of each satellite contains circular adjacent cells corresponding to the satellite spot-beams.

Depending on a coverage concept, two kinds of mobile satellite constellation systems can be defined: Satellite-Fixed Cells (SFC) and Earth-Fixed Cells (EFC) systems. This paper only focuses on SFC systems.

In the EFC coverage, satellites are able to steer their antennas in such a way that each beam maintains the coverage of a given earth-fixed cell during a given time duration [12]. The SFC coverage means that the cells described on the earth surface by each satellite beam are fixed to the spacecraft and move relatively to the earth surface. Consequently, and given that LEO satellites travel at a high velocity ($5 - 9km/s$), both the earth rotation and mobile users motion become negligible. Therefore, in this kind of systems, mobile and fixed users are treated in the same way as regards to the handover procedure. Besides, the number of handovers is function of both the satellite speed and the size of the cells corresponding to the spotbeams (in this study, we only focus on predictable handovers which are introduced by satellite motion and not those which result from shadowing, fading, and blocking effects). This number becomes very important especially in LEO satellite networks, leading to a real need of special CAC techniques and sophisticated handover management schemes.

To address this handover problem, a guaranteed handover scheme (GH) has been proposed in [12], [9]. This scheme guarantees to GH users (prioritized users) the success of all their handovers. In the following section, we give a brief description of the scheme and highlight its main drawbacks.

A. Guaranteed Handover scheme (GH)

The GH scheme has been proposed to hold the handover issue in LEO satellite systems supporting an SFC coverage. Two kinds of users are defined, prioritized users known as GH users and the other ones, called Regular users, do not benefit from the reservation strategy, and are not protected against handover fails.

The GH scheme tries to reserve a channel in the cell next to the one the user is entering. If such a channel is available it is locked, otherwise, a reservation request is sent waiting for a free channel. As the reservation is issued one cell before the user performs his handover, the success of this handover is guaranteed under the assumption that all the cells dispose of the same channel capacity C and also that the queued reservations have priority over both new calls (either GH or Regular) and handed over calls of Regular users. Concerning the case of a new generated call, it can be admitted in the system only if simultaneously two channels are idle in the first two cells (the source cell and the first transit cell). If ever one or both channels are not available, the call is blocked at setup.

The channel reservation used in this scheme is called Channel Locking mechanism. This strategy is in some manner very conservative and selfish, since a locked channel cannot be used by another user except the owner, even if the owner is not using it and is still far enough from the cell. To illustrate such a scenario, let us assume that an active GH user is performing his handover from cell $(i-1)$ to cell (i). At the same time, this GH user will lock a channel belonging to cell $(i+1)$ to guarantee the success of his handover to this cell. This channel will be locked during all the time necessary for the user GH to cross cell (i). Due to this "early" locking mechanism, a new generated call in cell (i) cannot be admitted (we assume that all the channels are used or locked in this cell) even if this user will leave cell (i) before the GH user arrives.

Consequently, this conservative locking strategy introduces a bad channel resource utilization when performing unnecessarily new calls blocking. It also results in an excessive prioritization of GH users which is achieved at the expense of a higher blocking probability of new call arrivals.

III. Time-based Channel Reservation Algorithm (TCRA) description

The Time-based Channel Reservation Algorithm (TCRA) is proposed to improve the GH performance and to provide a better resource utilization of the communication system. In this scheme, the channels are locked only for their expected time of use. This methodology allows to perform more accurate reservations in order to increase the number of admitted users in the system and to enhance the satisfaction degree of the users waiting for admission. However, it is worth stressing that this strategy can be achieved thanks to the deterministic and predictable satellite motion.

A. Basic assumptions and user mobility model

In this study, we are interested in two different QoS parameters : new call blocking and handover call dropping probabilities. We propose the following model which allows to derive these two performance parameters.

As said in the previous section, due to the high satellite velocity, the mobile users motion and the earth rotation speed are neglected. Therefore, users motion is straight and opposite to the satellite velocity vector. The system coverage geometry is illustrated in figure 1. Satellite spotbeams describe on the earth surface overlapping adjacent cells. Each cell is modeled as a rectangular area bounded by the segments joining intersection points of adjacent circular cells belonging to the same street of coverage. The side of each rectangular cell is referred to as the constant R. Let us assume that the entire bandwidth resource of each cell is divided into a fixed number of channels. Let C be this channel capacity.

B. Algorithm description

In this scheme, the aim is to compute time intervals necessary for a user to cross each cell belonging to the set of visited cells. These time intervals are used to reserve, in each of the considered cells, a channel which will be available during the corresponding crossing time duration. To implement such a method, each satellite should register, for each channel, all time periods where the channel is locked.

The proposed algorithm consists of three different phases:

Phase 1 : Call admission.

At call set up time T_{setup}, a channel reservation request is sent to the first two cells to be visited by the user: the source cell C_0, where the call was originated, and the first transit cell C_1.

Let T_i be the expected residence time of a user in a given cell C_i. In the source cell, T_0 is a variable uniformly distributed between 0 and R, whereas in transit cells, T_i (for $i > 0$) has a constant value T_{max} equal to (R/V_{sps}), where V_{sps} is the sub-satellite point speed.

In this study, we assume that the users locations can be determined, with a sufficient accuracy, since it is expected that either mobile or fixed terminals to be used in these systems would integrate positioning facilities such as Global Positioning System (GPS) receivers (otherwise, an other version of TCRA has been proposed in a previous work, the reader can refer to [2]).

At this step, the exact user location in the source cell is evaluated by the network, and the value of T_0 is derived.

Given this value, a reservation request is sent to cells C_0 and C_1 to reserve in each a channel for respectively the time intervals:

$$[T_{setup}, T_{setup} + T_0 + \delta t]$$

and

$$[T_{setup} + T_0 - \delta t, T_{setup} + T_0 + T_{max} + \delta t],$$

where δt is used to allow for a given error margin ($\delta t > 0$). If both requests are satisfied, the call is accepted, otherwise it is rejected.

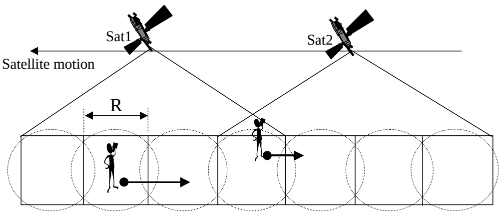

Fig. 1. User mobility model

This call admission procedure, when reserving in the first two cells, limits the number of admitted new calls in the system so that they do not cause a handover failure to any call in progress and also do not experience any handover failure. Therefore, if no blocking occurs at call setup, no handover failure is expected in the future since that the users relative geographical position remains the same, and only a time-lag will occur. Of course, this condition is verified under the assumption of a similar cell shape and also a similar capacity in terms of bandwidth (resources).

In other words, to simplify the proposed model, we have translated it into a one-dimensional problem by considering the transit time intervals of the users in each cell. Therefore, we only have to verify that the number of overlapping time intervals does not exceed the cell capacity C. This condition is necessary and sufficient to affirm that, at each instant, when considering the worst case where all the users are still active, there will not be more than C users under the same satellite beam coverage.

Phase 2: At each handover instant.

When a given user performs a handover from cell C_i to cell C_{i+1} at time T_{HOi}, the system can anticipate the future handover instant and thus reserve a channel in the upcoming cell C_{i+2} for the time interval

$$[T_{HOi}, T_{HOi} + T_{max} + \delta t].$$

Phase 3: Call termination.

When a user terminates its call in a cell C_i, it releases the current used channel and sends a reservation cancellation request to cell C_{i+1}.

IV. Analytical evaluation of the proposed scheme

A. Presentation of the method

In order to show the influence of the proposed mechanism on the performance of the system, an approximate analytical model has been performed in the case when only prioritized users P are considered. An analytical model has also been developed for Guaranteed Handover scheme.

Classical traffic assumptions are considered. New calls are assumed to arrive according to a Poisson process with parameter λ_{nc}. Uniform traffic is considered: all the cells are assumed to be offered the same new traffic intensity ; the residence time T_0 of a user in its original cell is assumed to be uniformly distributed between 0 and T_{max}. In the following cells, this time will be equal to T_{max}. Call durations, T_c, are assumed to be exponentially distributed with a parameter μ. Consequently, a new call will be taken into account in its originating cell during a time T_{nc} which is the minimum between T_c and T_0. In the following cells, due to the memoryless property of the exponential distribution, this residence time T_{ho} is the minimum between T_c and T_{max}.

$$\begin{cases} T_{nc} = inf(T_c, T_0) \\ T_{ho} = inf(T_c, T_{max}) \end{cases}$$

The expectation of these r.v. can easily be derived:

$$E[T_{nc}] = \frac{1}{\mu} - \frac{1 - e^{-\mu T_{max}}}{\mu^2 T_{max}}, \; E[T_{ho}] = \frac{1 - e^{-\mu T_{max}}}{\mu}$$

The call admission control can be expressed as follows (parameter δt is neglected as in [9]). Let $N_t(y)$ denote the number of users at time t with an abscissa between y and $(y + R)$. This area covers a part of two consecutive cells (see figure 2). If a given user arrives at time t with an initial abscissa x, $0 \leq x \leq R$, this new call will be accepted iff

$$\forall y, \qquad x - R \leq y \leq x, \qquad N_t(y) < C \tag{1}$$

This formula obviously concerns cell C_0, C_{-1} and C_1.

In the case when only prioritized traffic is considered, GH algorithm can be described as follows. A new call will be accepted if less than C channels are occupied or locked in cell C_0 and C_1. As each active user in cell C_{-1} (resp. C_0) has locked a channel in cell C_0 (resp. C_1), the CAC condition leads to:

$$(N_t(-R) + N_t(0) < C) \text{ and } (N_t(0) + N_t(R) < C) \tag{2}$$

(2) is equivalent to

$$\forall y, \qquad -R \leq y \leq R, \qquad N_t(y) < C \tag{3}$$

As this condition includes the previous one, GH is more restrictive than TCRA. Both solutions leads to a handover dropping probability equal to 0, but TCRA improves the performance for new calls.

An exact model of the whole system is quite complicated to be derived because it is necessary to know the number of in-progress calls in each cell and their relative positions. Consequently, an approximate model has been developed. Classical approximations are proposed. The handover arrival process is approximated by a Poisson process with parameter λ_{ho} which parameter has to be computed. In the model, the users are also supposed to be uniformly distributed over all the cell. An isolation method is proposed which consists on considering independence between the cells [9].

Under those approximations, a cell is modeled by a multiclass M/G/C/C queue with reservation. A first class corresponds to the actual number i of "new calls" (i.e. those initiated in the current cell) and a second class to the number j of "handover calls" (i.e. those initiated in a previous one). Let p_k be the new call blocking probability when k resources are occupied by a new call or a handover. The accepted new call arrival rate is then equal to $\lambda_{nc}(1-p_k)$. We used the numerical solution of the steady state distribution of the corresponding multiclass M/M/C/C queue with reservation [3] with respective service rates:

$$\mu_{nc} = \frac{1}{E[T_{nc}]}, \qquad \mu_{ho} = \frac{1}{E[T_{ho}]}$$

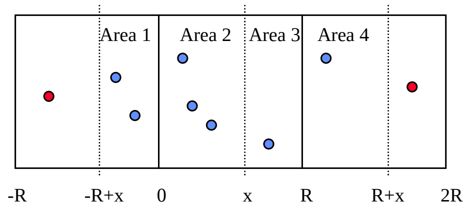

Fig. 2. Derivation of the blocking probability

Let $\pi_{i,j}$ denote the steady state probability of state (i,j) and Π_k the marginal probability of having k occupied resources. The same approach has been adopted to analyse both GH and TCRA mechanisms. The main difference comes from the derivation of p_k.

B. Blocking probabilities for TCRA mechanism

Given the uniform position of the users and the uniform arrivals of users within a cell, p_k can be derived as follows (we only consider the steady state values). Let Y denote the initial offset of a new call arrival, and V the actual configuration when a new call arrives:

$$p_k = \frac{1}{R}\int_{x=0}^{R} \sum_{l,m=0}^{C} Pr[\text{New Call blocked}|\mathcal{V}]\Pi_l\Pi_m dx$$

with

$$\mathcal{V} = \{Y = x, N_t(-R) = l, N_t(+R) = m, N_t(0) = k\}$$

(in configuration V, there are k users in cell C_0, l in cell C_{-1} and m in cell C_1).

This blocking probability depends on the number of users within the area $[x-R, x+R]$ (see Fig. 2). When a user arrives he finds a configuration which satisfies condition (1). It is necessary to determine among all the possible users positioning configurations those which are not blocking for the arrival of the new call. Computation details are presented in the Appendix.

The numerical solution of the Markov chain corresponding to the multiclass M/M/C/C queue with reservation leads then to the derivation of the new call blocking probability $P_{b,nc}$, PASTA property can be applied:

$$P_{b,nc} = \sum_{k=0}^{C} \Pi_k p_k$$

The handover rate λ_{ho} can then be derived. Let τ_{nc} (resp. τ_{ho}) the probability for an accepted new call (resp. a handover) to experience a handover (resp. a new handover).

$$\tau_{nc} = \frac{1-e^{-\mu T_{max}}}{\mu T_{max}}, \qquad \tau_{ho} = e^{-\mu T_{max}}$$

$$\lambda_{ho} = \lambda_{nc}(1-P_{b,nc})\tau_{nc} + \lambda_{ho}\tau_{ho}$$

which leads to

$$\lambda_{ho} = \frac{\lambda_{nc}(1-P_{b,nc})\tau_{nc}}{1-\tau_{ho}}$$

A recursive approach is then necessary to derive the blocking probability and the handover rate. The first iteration starts by neglecting the new call blocking probability

$$\lambda_{ho}^{0} = \frac{\lambda_{nc}\tau_{nc}}{1-\tau_{ho}}$$

The iterative method is stopped when the relative difference between the blocking probability values computed in two subsequent steps is below a threshold ϵ.

C. Blocking probabilities for GH mechanism

The derivation of the blocking probabilities p_k are easier to determine for GH mechanism using (2). They can then be written as follows:

$$1 - p_k = \sum_{l=0}^{C-k-1}\sum_{m=0}^{C-k-1} \Pi_l\Pi_m$$

which is simply equal to

$$1 - p_k = \left(\sum_{m=0}^{C-k-1} \Pi_m\right)^2$$

The derivation of the performance criteria can be obtained using the previous method.

V. Performance evaluation

The performance evaluation of both TCRA and GH schemes has been investigated through analytical and simulation results. Extensive simulation experiments have been carried out to validate the proposed analytical approach. The tests presented are aimed at showing the behavior of the proposed strategy TCRA, and at highlighting its advantages with respect to Guaranteed Handover scheme.

In particular, we have considered that the simulated cellular network is a grid of 36 square shaped cells folded onto itself. Each cell corresponds to a beam of the satellite. The users are assumed to cross the cellular network with a constant relative velocity orthogonal to the side of the spotbeams. The model considers only one class of users, prioritized users P, which benefit from reservation strategies of both GH and TCRA schemes. Moreover, a fixed channel allocation (FCA) technique has been used for the allocation of satellite channels to beams (cells). We describe, in the following, the main parameter values used in the simulated scenario :

• New call arrivals in a given cell are assumed to be Poisson processes, with a channel holding time exponentially distributed.
• The communication's lifetime of the users is exponentially distributed. The mean call duration is fixed to 180 seconds.
• The number of available channels per beam is 20.
• V_{sps} and R are fixed respectively to 25.000 km/h and 250 km.

The model allows to generate call blocking probabilities of the P users using GH and TCRA schemes. The handover dropping probability is not plotted in the figure since it shows a null value with both schemes.

Intuitively, as TCRA tries to reserve the resources for only the expected time of their use, it yields to a shorter locking duration. Hence, the resources are more available for new arriving users which have a greater chance to be admitted in the system.

With GH scheme, the resources are locked before their effective use, so they reside in the locked state for longer time and block entry of more new calls increasing the blocking probability.

These tendencies are verified in figure 3. The illustrated curves show the analytic and simulation results for TCRA and GH schemes.

Firstly, we can easily note the good agreement between the results derived by the analytical method for both schemes and those obtained by simulations. The slight difference is exclusively due to the approximations of the analysis when assuming a Poisson distribution for handover requests.

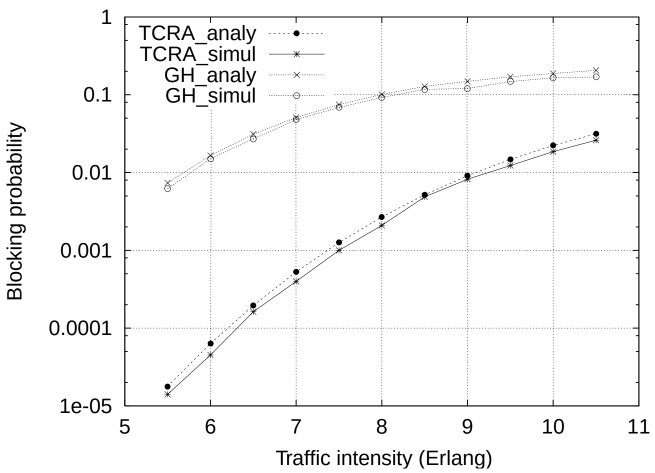

Fig. 3. Call blocking probabilities for TCRA and GH schemes

Changing traffic intensity by increasing λ_{nc}, the call blocking probability increases accordingly using both schemes. However, results show clearly that TCRA reduces significantly this blocking probability with respect to the GH scheme especially when dealing with low traffic loads. We can also remark that the reduction obtained by TCRA is about a mean factor of 100 for the traffic range under examination.

VI. Conclusions

This paper proposes a new time-based channel reservation scheme called TCRA for handover control and management in LEO mobile satellite systems, especially those supporting a satellite-fixed cell coverage. TCRA is based on the feature that, in LEO systems, the users mobility and trajectory are predictable. It guarantees to users a null handover failure probability during all their communication lifetime. It has the advantage of reserving channel resources for users only the expected time duration where they are supposed to be under the coverage of the considered beam. An analytical model has been developed for both TCRA and the Guaranteed Handover (GH) scheme. The results obtained analytically and by simulation point out that TCRA can achieve a better channel utilization than GH. The new call blocking probabilities have been reduced significantly leading to a higher satisfaction degree of the whole potential user population.

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Annex A

Let $\phi(C-n,i,j)$ denote the number of configurations with i users in area 1, j in area 2 and $n-i$ in area 3 (see Fig. 2) which satisfies

$$\forall y, \qquad x-R \leq y \leq 0, \qquad N_t(y) < C \tag{4}$$

Assuming a uniform distribution of the users in the cells, it can easily be found that this number does not depend on the value x and consequently that this formula can also be applied to areas 2, 3 and 4. This leads to:

$$1-p_k = \sum_{l=0}^{C}\sum_{m=0}^{C}\Pi_l\Pi_m\sum_{l'=0}^{l}\sum_{m'=0}^{m}\sum_{k'=0}^{k}\Phi\int_{x=0}^{R}\frac{G}{R}dx \tag{5}$$

where

$$\Phi = \frac{\phi(C-k+k'-l'-1,l',k')}{\phi(C-k+k'-l',l',k')}\frac{\phi(C-k-1,k-k',m')}{\phi(C-k,k-k',m')}$$

and

$$G = \begin{pmatrix} k \\ k' \end{pmatrix}\begin{pmatrix} l \\ l' \end{pmatrix}\begin{pmatrix} m \\ m' \end{pmatrix}\left(1-\frac{x}{R}\right)^I\left(\frac{x}{R}\right)^J$$

with $I=(l'+m-m'+k')$ and $J=(l-l'+m'+k-k')$

As,

$$\frac{1}{R}\int_{x=0}^{R}(1-\frac{x}{R})^n(\frac{x}{R})^p dx = \frac{n!p!}{(n+p+1)!}$$

equation (5) may be simplified as follows:

$$\int_{x=0}^{R}\frac{G}{R}dx = \begin{pmatrix} k \\ k' \end{pmatrix}\begin{pmatrix} l \\ l' \end{pmatrix}\begin{pmatrix} m \\ m' \end{pmatrix}\frac{I!J!}{(I+J+1)!} \tag{6}$$

$\phi(K,i,j)$ can be recursively computed.

$$\phi(K,i,j) = i\phi(K+1,i-1,j) + j\phi(K-1,i,j-1)$$
$$K \geq 0, i > 0, j > 0$$

The bounds are obtained as follows:

$$\begin{cases} \phi(-1,i,j) & = & 0 & \text{if } i > 0 \text{ or } j > 0 \\ \phi(-1,0,0) & = & 1 & \\ \phi(K,i,j) & = & (i+j)! & \text{if } K \leq j \end{cases}$$

It finally leads to the derivation of the parameters p_k.